Entering Name: ________________________________ Summer 2023

Calculus Summer Packet Mrs. Foltz

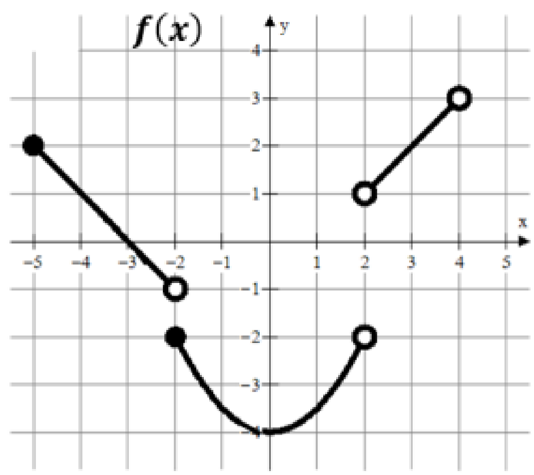
<u>The Packet:</u> Calculus constantly requires students to use their prerequisite knowledge from Algebra and Pre-Calculus. That being said, it is critical to enter this class with a solid foundation in major topics taught in these courses. This packet will review these major concepts that are often used in Calculus, as well as introduce students to the first topic taught in Calculus: *Limits and Continuity*.

<u>Directions:</u> This packet is to be completed by your first day of the school year. Show all work in the packet or on a separate sheet of paper attached to the packet. Completion of this packet will be counted as your first grade of Quarter 1.

Evaluating Functions (Algebraically): Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$	**2.** $f(x+2) =$	**3.** $f(x+h) =$

Evaluating Functions (Graphically): Use the graph of $f(x)$ find the following.

<table>
<tr><td>4. $f(0) =$</td><td>5. $f(4) =$</td><td rowspan="4">
</td></tr>
<tr><td>6. $f(-1) =$</td><td>7. $f(-2) =$</td></tr>
<tr><td>8. $f(2) =$</td><td>9. $f(3) =$</td></tr>
<tr><td>10. $f(x) = 2$ when $x = ?$</td><td>11. $f(x) = -3$ when $x = ?$</td></tr>
</table>

Rational Exponents: Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$.

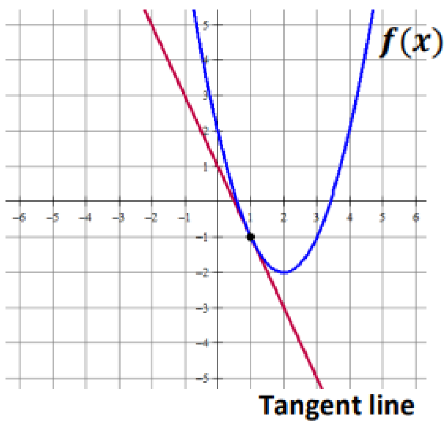
12. $\frac{3}{\sqrt{x}}$	**13.** $\sqrt{x+1}$	**14.** $\frac{1}{\sqrt{x}} - \frac{2}{x}$

Writing Linear Equations Given Conditions: Write the equation of the line that meets the following conditions. Use point-slope form: $y - y_1 = m(x - x_1)$.

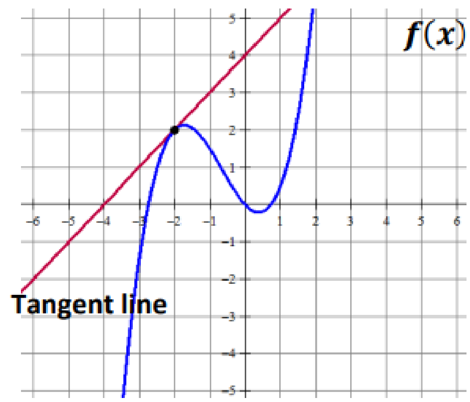
15. Slope of 3 and passes through point $(4, -2)$	**16.** $m = -\frac{3}{2}$ and $f(-5) = 7$	**17.** $f(4) = -8$ and $f(-3) = 12$

Writing Equations of Tangent Lines: Write the equation of the tangent line in point-slope form.

18. The line tangent to $f(x)$ at $x = 1$



19. The line tangent to $f(x)$ at $x = -2$



Slope of Secant Lines: Determine which choice represents the slope of the secant line shown using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

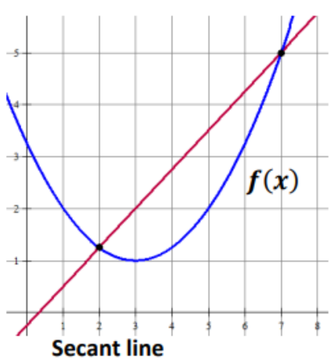
20. Circle one answer below.

(A) $\frac{7-2}{f(7)-f(2)}$

(B) $\frac{f(7)-2}{7-f(2)}$

(C) $\frac{7-f(2)}{f(7)-2}$

(D) $\frac{f(7)-f(2)}{7-2}$



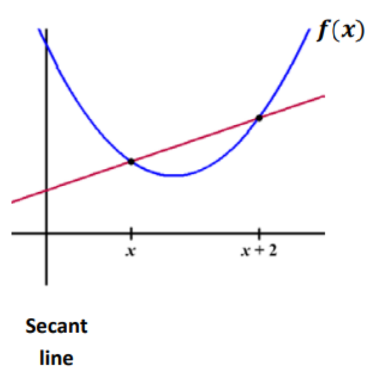
21. Circle one answer below.

(A) $\frac{f(x)-f(x+2)}{x+2-x}$

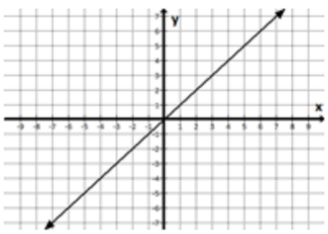
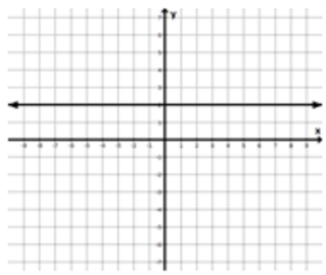
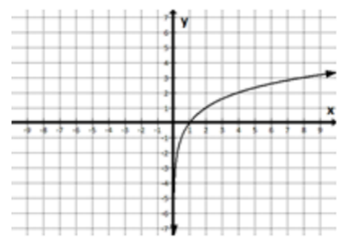
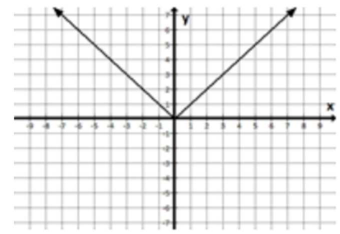
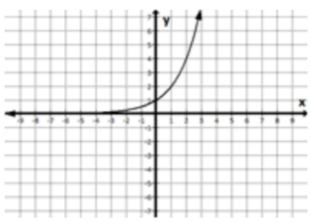
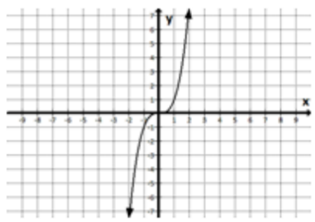
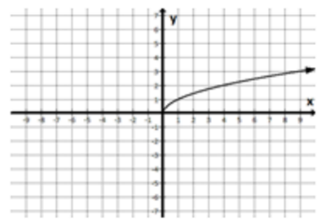
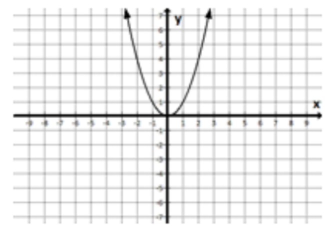
(B) $\frac{f(x+2)-f(x)}{x+2-x}$

(C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

(D) $\frac{x+2-x}{f(x)-f(x+2)}$



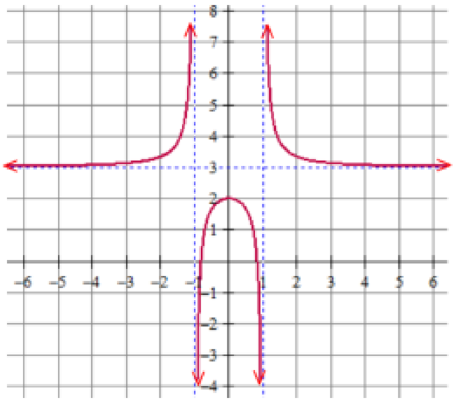
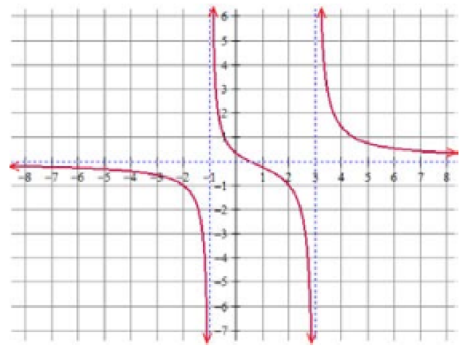
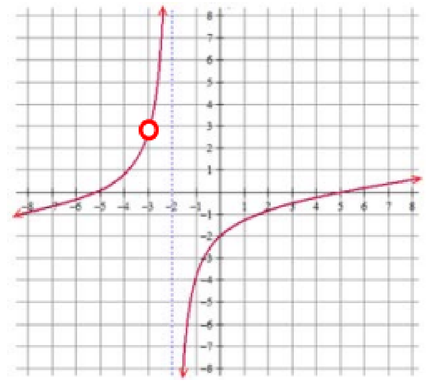
Parent Functions: It is important to recognize parent functions. Label each function below with one of the following functions: *Constant, linear, quadratic, cubic, absolute value, natural log (lnx), exponential (e^x), radical (square root).*

22. 	**23.** 	**24.** 	**25.** 
26. 	**27.** 	**28.** 	**29.** 

Domain & Range: For each function, determine its domain and range (express in interval notation).

30. $y = \sqrt{x - 2}$ Domain: Range:	**31.** $y = (x - 3)^2$ Domain: Range:	**32.** $y = ln(x)$ Domain: Range:
33. $y = e^x$ Domain: Range:	**34.** $y = \sqrt{4 - x^2}$ Domain: Range:	**35.** $y = x^4 + 3x^2 - 7$ Domain: Range:

Rational Functions (Graphically): Find the domain and range (express in interval notation), as well as all horizontal and vertical asymptotes.

36.	**37.**	**38.**
 Domain: Range: Horizontal Asymptote(s): Vertical Asymptote(s):	 Domain: Range: Horizontal Asymptote(s): Vertical Asymptote(s):	 Domain: Range: Horizontal Asymptote(s): Vertical Asymptote(s):

Rational Functions (Algebraically): Determine all vertical asymptotes, horizontal asymptotes, and holes.

39. $f(x) = \frac{1}{x^3}$	**40.** $f(x) = \frac{x^2}{x^2-4}$	**41.** $f(x) = \frac{x^2-2x+1}{2x^2-6x+4}$
Horizontal Asymptote(s): Vertical Asymptote(s): Hole(s):	Horizontal Asymptote(s): Vertical Asymptote(s): Hole(s):	Horizontal Asymptote(s): Vertical Asymptote(s): Hole(s):

Exact Value Trigonometry: You need to know basic trig functions in RADIANS! We never use degrees in Calculus. It is helpful to be familiar with the unit circle. Evaluate the given expression (no calculator).

42. $sin\left(\frac{\pi}{6}\right) =$	**43.** $cos\left(\frac{\pi}{4}\right) =$	**44.** $sin(2\pi) =$	**45.** $tan(\pi) =$
46. $sec\left(\frac{\pi}{2}\right) =$	**47.** $cos\left(\frac{\pi}{6}\right) =$	**48.** $sin\left(\frac{\pi}{3}\right) =$	**49.** $sin\left(\frac{3\pi}{2}\right) =$
50. $tan\left(\frac{\pi}{4}\right) =$	**51.** $csc\left(\frac{\pi}{2}\right) =$	**52.** $sin(\pi) =$	**53.** $cos\left(\frac{\pi}{3}\right) =$

Inverse Trigonometry: Recall the inverse trigonometry is just the reverse of what you did above. Also, it can be written as, for example, $arctan(x)$ or $tan^{-1}(x)$. Evaluate the given expressions (no calculator) over the interval $[0,\ 2\pi]$. All answers are in RADIANS. Example: Since $sin(\pi) = 0$, then $sin^{-1}(0) = \pi$.

54. $cos^{-1}(0) =$	**55.** $arcsin\left(\frac{1}{2}\right) =$	**56.** $tan^{-1}(-1) =$	**57.** $arcsin(1) =$

Complex Fractions: Simplify each of the following (there should be no fractions within fractions).

58. $\frac{\frac{25}{a}-a}{5+a}$	**59.** $\frac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$	**60.** $\frac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

Solving Logarithmic & Exponential Equations: Solve the following equations for x.

61. $e^{x} + 1 = 2$	**62.** $3e^{x} + 5 = 8$	**63.** $e^{2x} = 1$
64. $e^{2x} - e^{x} = 0$	**65.** $ln(x) = 0$	**66.** $3 - ln(x) = 3$

Composite Functions: Let $f(x) = 2x + 1$ and $g(x) = x^2 - 1$. Evaluate each of the following.

67. $f(g(1))$	**68.** $g(f(-2))$	**69.** $f(g(x^3))$

Simplifying Expressions: Simplify the following using logarithmic and exponential rules.

70. $ln(1)$	**71.** $ln(e^7)$	**72.** $log_3\left(\frac{1}{3}\right)$
73. $log_{1/2}8$	**74.** $log\left(\frac{1}{2}\right)$	**75.** $27^{\frac{2}{3}}$
76. $\left(5a^{\frac{2}{3}}\right)\left(4a^{\frac{3}{2}}\right)$	**77.** $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$	**78.** $\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$

This page is required for students entering AP Calculus only, but recommended to all students entering Calculus. *Limits* is a major part of the first unit.

Limits: Evaluate each limit, if it exists, algebraically (without a graphing calculator). Remember to always start with direct substitution. If direct substitution results in indeterminate form, try to factor/simplify or rationalize the fraction. For one-sided limits, make sure to consider the behavior of the graph, including any holes or asymptotes.

Scan the QR Code for review on limits if you are unfamiliar or struggling with the concept

https://youtu.be/CSALbevEMfs

79. $\lim_{x \to 2} (4x^2 + 3)$	**80.** $\lim_{x \to 1} \frac{x^2+x+2}{x+1}$	**81.** $\lim_{x \to \pi} cos(x)$
82. $\lim_{x \to 1} \frac{x^2-1}{x-1}$	**83.** $\lim_{x \to -3} \frac{x^2+x-6}{x+3}$	**84.** $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$
85. $\lim_{x \to 3} \frac{3-x}{x^2-9}$	**86.** $\lim_{x \to 5^+} \frac{x-5}{x^2-25}$	**87.** $\lim_{x \to 4^-} \left(-\frac{3}{x-4}\right)$